



ORIGINAL ARTICLE

Effects of van Hiele Theory-based Instruction on Pre-service Mathematics Teachers' Geometry Knowledge, Geometric Thinking Skills and Attitudes: A Literature Review

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ABSTRACT

Teachers' geometry competencies are key factors determining the effectiveness of students' geometry learning. This paper employs a comprehensive research and content analysis to examine the application of van Hiele theory in developing geometry competencies, encompassing geometry knowledge, geometric thinking skills and attitudes, among pre-service mathematics teachers. Research findings demonstrate that organizing teaching around van Hiele's five-stage process enhances pre-service mathematics teachers' knowledge of geometric content and teaching geometry, improves their geometric thinking skills, and strengthens positive attitudes towards teaching and learning geometry. The study also emphasizes the role of technological tools in encouraging pre-service mathematics teachers to actively engage in the learning stages according to the van Hiele model. Based on these results, the paper offers recommendations for teacher training programs to improve the quality of geometry teaching by pre-service mathematics teachers, focusing on both content knowledge and effective teaching methods.

1. INTRODUCTION

As an important area of mathematics, geometry is closely related to other areas and widely applied in practice (Yi et al., 2020). However, research has increasingly highlighted students' struggle with learning geometry (Yi et al., 2020), and teachers' limited competence in teaching geometry (Aslan-Tuktak & Adams, 2015). Previous studies have also emphasized the importance of teaching quality in improving students' geometry learning outcomes (Bulut & Bulut, 2012; Mbusi & Luneta, 2023). As a result, there have been numerous efforts to enhance pre-service teachers' geometry competencies through training and professional development programs, focusing on developing teachers' geometric content knowledge, geometric thinking skills, and geometry teaching skills (Aslan-Tuktak & Adams, 2015; Mbusi & Luneta, 2023; Yi et al., 2020). In the constructivist approach, while teachers are responsible for helping students to attain content knowledge beyond memorization and construct their own content knowledge, they are viewed as a person possessing specialised and sufficient content knowledge (van der Sandt, 2007). Noticeably, van Hiele theory provides suggestions for both the improvement of pre-service teachers' geometric thinking and teaching practice (Yi et al., 2022).

Accordingly, a common approach to training pre-service mathematics teachers is the application of van Hiele's theory in teaching geometry content to pre-service mathematics teachers, with the aim of improving pre-service teachers' geometric competencies and understanding of students' geometry learning process (Erdoğan & Durmuş,

2009; Yi et al., 2020). Though there has been no commonly accepted conceptualization of the term competency, it can be synthesized that competency encompasses three facets, including knowledge, skills and attitudes (Ala-Mutka, 2011). In line with this definition, Dubois (1998) and Teodorescu (2006) considered competencies as knowledge, skills, mindsets, thought patterns, which, when used either singularly or in various combinations, result in successful performance. Similarly, the combination of teachers' knowledge, skills, understanding, values, and attitudes is also mentioned when examining teachers' competencies (Verner et al., 2019). Drawing on these definitions, this current study explores pre-service teachers' geometry knowledge, geometric thinking skills and attitudes towards geometry when examining the influence of van Hiele theory-based instruction on pre-service teachers competencies. The term 'knowledge' refers to familiarity with factual information and theoretical concepts (European Parliament Council, 2008), while 'skills' pertains to the ability to apply knowledge to specific situations, and 'attitudes' are conceived as motivators of performance and include ethics, values, and priorities as well as responsibility and autonomy (Ala-Mutka, 2011).

These competencies are gained from teachers' practical experience through reflection and conceptualization (Verner et al., 2019). Previous studies suggest that during professional development, pre-service teachers should acquire knowledge-in-practice, knowledge-for-practice, and knowledge-of-practice (Cochran-Smith & Lytle, 1999). Therefore, it is important for teacher training courses to actively involve pre-service teachers in learning about both the content and methods of teaching geometry (Garet et al., 2001). van Hiele's theory provides a model of geometric thinking levels and corresponding learning stages (van Hiele, 1986), which helps pre-service mathematics teachers connect their own geometric knowledge to the content they need to teach and the effective teaching methods (Banson et al., 2023). The model categorizes geometric thinking into five levels: visualization, analysis, informal deduction, formal deduction, and rigor (Fuys et al., 1988; van Hiele, 1986; van Hiele, 1999). According to this model, students must engage in activities that explore the characteristics of geometric concepts (Erdoğan & Durmuş, 2009) through five learning stages that promote students' progress from one van Hiele level to the next, including information, directed orientation, explication, free orientation, and integration (Fuys et al., 1988; van Hiele, 1986; van Hiele, 1999).

The application of van Hiele's theory to pre-service mathematics teacher education, therefore, comprises two main aspects, namely guiding teaching activities, and providing a theoretical framework for designing assessments (Yi et al., 2020). The effectiveness of this training approach in developing geometry competencies of pre-service mathematics teachers has been examined in numerous previous studies, concerning the aspects of geometric content knowledge, geometric thinking skills, and attitudes towards geometry (Yi et al., 2022). Given the significant body of literature on the application of the van Hiele model in developing geometry competencies for pre-service mathematics teachers (Çelik & Yılmaz, 2025; Hassan et al., 2020), a comprehensive review of this topic is necessary to provide an overview of relevant aspects. Particularly, there is still a gap for a comprehensive review that is specifically focused on the impacts of van Hiele theory-based instruction on improving the geometry competencies of pre-service mathematics teachers regarding their geometry knowledge, geometric thinking skills, and attitudes towards teaching and learning geometry. Therefore, this study aims to bridge that gap, seeking answers to the following research question:

How does the application of the van Hiele theory-based instruction affect geometric knowledge, geometric thinking skills, and attitudes towards geometry of pre-service mathematics teachers?

2. LITERATURE REVIEW

2.1. The van Hiele model of geometric thinking levels

The van Hiele model of geometric thinking levels (van Hiele, 1986, 1999) has been widely used in mathematics education. This model posits that geometric thinking encompasses five levels: Visualization, Analysis, Informal deduction, Formal deduction, and Rigor. At the Visualization level, students can identify geometric shapes based on their appearance; at the Analysis level, students can examine shapes through their properties; at the Informal deduction level, students begin to establish relationships between the properties of a shape and between shapes; at the Formal deduction level, students can perform proofs, understand the role of axioms, definitions, necessary and sufficient conditions; and at the Rigor level, students can understand and compare axiomatic systems in geometry.

According to this theory, levels of geometric thinking are sequential and hierarchical, requiring students to progress from lower to higher levels (van Hiele, 1986; Verner et al., 2019). Furthermore, each level has its own distinct language system, symbols, and associations, and student progress depends on the appropriateness of the teacher's instructional activities to students' levels of geometric thinking (Clements & Battista, 1992). Therefore, teachers need to apply teaching methods that align with students' levels of geometric thinking, using content and activities that promote the development of their geometric thinking (Yi et al., 2020).

2.2. Learning stages based on the van Hiele model

Accordingly, van Hiele's theory proposes learning stages based on the van Hiele model – a theoretical framework for teaching geometry aimed at promoting students' understanding and progress in the van Hiele levels of thinking (Ndungo et al., 2025). This theoretical framework guides teachers in designing lessons with activities aligned with the levels of thinking, helping students progress to higher levels through active learning and teacher guidance (Mbusi & Luneta, 2023; Ndungo et al., 2025). There are five learning stages according to the van Hiele model: Information, Directed Orientation, Explication, Free Orientation, and Integration (Fuys et al., 1988). Students must go through the Information, Guided Orientation, Explication, Free Orientation, and Integration phases to advance from the first level to the second level, and then they have to go through the same phases to advance to the next stages (Abdullah & Zakaria, 2013; Ndungo et al., 2025). These stages are described in Table 1 (Fuys et al., 1988; Howse & Howse, 2015; Mbusi & Luneta, 2023; van Hiele, 1986).

Table 1. Learning stages according to the van Hiele model

Stage	Description
Information	The teacher introduces the learning content. Learners become familiar with vocabulary and concepts; The teacher evaluates learners' explanations to determine the following tasks.
Directed orientation	Learners actively participate in activities under teacher guidance; Learners develop results from previous tasks to better understand the nature of concepts and their relationships.
Explication	Learners recognize initial connections, analyze them based on existing knowledge, and verbally articulate the concepts; The teacher explains the concepts and standardizes geometric language.
Free orientation	The teacher provides more complex learning activities for learners to complete independently. Learners choose ideas and formulate their own framework of connections.
Integration	Learners summarize, integrate, reflect on, describe, and apply the knowledge they have learned.

As presented in existing literature, Van Hiele's theory has been used in teaching geometry content in various teacher-training programs for pre-service mathematics teachers (Armah & Kissi, 2019). These programs not only provide specialized knowledge to pre-service mathematics teachers but also aim to develop their ability to design effective teaching methods using the van Hiele model (Armah & Kissi, 2019). In particular, the van Hiele's theory is applied in teacher education to cultivate geometry teaching competence of pre-service teachers through two forms: Firstly, it guides teaching activities aimed at forming and developing geometric competencies and the ability to teach geometry for pre-service mathematics teachers (Yi et al., 2020); Secondly, the van Hiele model serves as a theoretical framework for designing assessments to evaluate pre-service mathematics teachers' understanding of students' levels of geometric thinking and the corresponding geometry teaching activities for each level of geometric thinking (Yi et al., 2020).

3. MATERIALS AND METHODS

Using a comprehensive research and content analysis approach, this paper aims to systematize the application of the van Hiele model in pre-service mathematics teacher education and its impact on fostering their geometry teaching competence. The content analysis method used in this study is considered a systematic approach for evaluating trends and research results in a descriptive direction (Çalık & Sözbilir, 2014). The study by Çelik and Yılmaz (2025) on the application of the van Hiele model across nations worldwide also employed this method.

In this review, we referred to the PRISMA guidelines (Moher et al., 2010) to address the above research questions. The authors specifically selected and analyzed relevant studies published within the last 10 years (2016–December 2025). The databases used for the research included ERIC, Google Scholar, and Scopus, all popular sources that provide easy access to scientific publications. The research searches in each database were conducted using keywords such as (“pre-service mathematics teachers” or “prospective mathematics teachers”), (“van Hiele”), (“geometry competencies” or “knowledge” or “thinking skill” or “attitudes”), and (“teacher training” or “teacher education”), along with synonyms and required search syntax.

The selection of studies for this review was based on the following inclusion criteria: they were English-language articles or conference reports, published between 2016 and 2025, empirical studies focusing on the application of the van Hiele theory-based instruction in training pre-service mathematics teachers, and examining the effect of the van Hiele theory-based instruction on pre-service mathematics teachers’ geometry knowledge, geometric thinking skills, or/and attitudes. Studies that did not meet the inclusion criteria would be excluded. The selected studies were then included or excluded by manual assessment based on screening their titles, keywords, and abstracts, followed by full-text review. The authors assessed these studies independently before discussions for consensus to avoid bias. As a result, 15 relevant studies were selected as the dataset for this review.

The analyses were conducted by extracting research results on the impact of applying the van Hiele theory-based instruction in teacher training on pre-service mathematics teachers’ geometry knowledge, geometric thinking skills, and attitudes towards geometry and geometry teaching.

4. RESULTS AND DISCUSSION

Review studies on van Hiele’s theory show its widespread application in mathematics teacher training to improve the effectiveness of teaching and learning geometry for pre-service teachers (Armah & Kissi, 2019). Specifically, the use of van Hiele’s theory in developing aspects of geometry competencies for pre-service mathematics teachers, such as geometric content knowledge, geometric thinking skills, and attitudes towards teaching and learning geometry, has been investigated in many previous studies with diverse results (Table 2).

Table 2. Statistical results of some relevant studies

No.	Study	Purpose	Topic	Research Methodology	Sample	Instruments	Intervention duration (if applied)	Result
1	Atta and Bonyah (2023)	Explore the impact on academic performance	Geometry in the plane	Quantitative method	101	Pre and post tests	3 weeks	Improving academic performance
2	Yi et al. (2022)	Explore the impact of geometry teaching knowledge	2D shapes	Quantitative methods	307	Tests	3 weeks	A proposed framework for assessing geometry teaching skills for pre-service mathematics teachers
3	Yi et al. (2020)	Explore the impact of teaching geometry knowledge	2D shapes	Quantitative method	111	Tests	3 weeks	Improving geometry teaching knowledge

4	Shayeb et al. (2024)	Explore the impact of geometric content	Diagonals of polygons	Qualitative design	23	Pre-questionnaires, post-questionnaires, and observations of class discussions	—	Significantly improved conceptual knowledge and conceptual images
5	Usman et al. (2023)	Investigate the effect of van Hiele's geometric model on pedagogical abilities of undergraduate mathematics education teachers	—	Quantitative methods	100	Geometry Teaching Practice Assessment	—	Geometric pedagogical abilities of undergraduate mathematics education teachers were positively improved
6	Nopriana et al. (2023)	Examine the impact on the development of geometric thinking	—	Descriptive quantitative research method	48	Questionnaire, van Hiele geometry test	—	Enhancing geometric thinking abilities using the van Hiele model
7	Yilmaz and Koparan (2016)	Explore the impact on the development of geometric thinking	2D shapes	Mixed methods	44	Van Hiele Geometry Test, interview	14 weeks	Enhancing geometric thinking abilities using the van Hiele model
8	Mbusi and Luneta (2023)	Using van Hiele's theory to overcome misconceptions	Transformation	Mixed methods	82	Field notes, observations, informal conversations, and test	3 months	Identifying and effectively correcting misconceptions
9	Mbusi and Luneta (2021)	Using van Hiele's theory to overcome misconceptions	Transformation	Qualitative interpretive study	82	Tests, interview	—	Identifying and effectively correcting misconceptions
10	Mbatha and Bansilal (2023)	Using van Hiele's theory to overcome misconceptions	—	Qualitative method	34	Worksheet, semi-structured interview	—	Identifying and effectively correcting misconceptions
11	Erdogan (2020)	Using van Hiele's theory to overcome difficulties in problem identification skills	The concept of analogy in Euclidean geometry	Qualitative method	65	Written documents	—	Improving problem-solving skills
12	Jupri (2018)	Using van Hiele's theory to improve geometric proofing skills	—	Qualitative method	9	Reasoning test, written works	—	Improving geometric proofing skills and deductive thinking
13	Usman et al. (2019)	Explore the impact on	—	Qualitative method	149	Questionnaire and tests	8 weeks	Promoting a positive attitude

		attitudes toward geometry					towards geometry
14	Banson et al. (2022)	Explore the impact on attitudes toward geometry	—	Descriptive and quantitative methods	225	Fennema-Sherman Attitude Scales, Geometry Attitude Scale, van Hiele Geometry test	—
15	Mohamed et al. (2025)	Combining with GeoGebra for active learning engagement	—	Qualitative method	17	Semi-interviews, classroom observations, GeoGebra task completion, digital objects, tests, and electronic surveys	2 months
							Enhancing interest, confidence, and awareness of the usefulness of geometry
							Pre-service mathematics teachers actively participate in learning activities

4.1. Impacts of van Hiele theory-based instruction on pre-service mathematics teachers' geometry knowledge

Previous studies have shown that the application of van Hiele's theory in teaching geometry content to pre-service mathematics teachers has led to improvements in geometry learning outcomes (Atta & Bonyah, 2023; Usman et al., 2019), geometry content knowledge (Shayeb et al., 2024; Yi et al., 2022), and geometry teaching knowledge of pre-service mathematics teachers (Yi et al., 2020; Yi et al., 2022). For example, regarding the potential impact of using van Hiele's learning stages in teaching plane geometry to pre-service mathematics teachers, studies have shown that their learning outcomes improve significantly (Atta & Bonyah, 2023). Similarly, Usman et al. (2019) confirmed the effectiveness of learning design grounded in van Hiele's theory in maintaining long-term learning outcomes for pre-service mathematics teachers in a quasi-experimental study involving nearly 150 participants. Additionally, van Hiele's theory has been used to teach a variety of geometry topics to help pre-service teachers build their geometric knowledge. Teaching geometry related to a plane (Atta & Bonyah, 2023), 2D shapes (Yi et al., 2022), and the diagonals of polygons (Shayeb et al., 2024) are part of this. These studies, through interventions that target learning stages according to van Hiele's thinking levels model, can help pre-service mathematics teachers reconstruct their definitions of geometric concepts and identify critical attributes of these concepts, thereby improving their ability to extend the concept image and develop their geometric knowledge and understanding (Shayeb et al., 2024).

Furthermore, applying van Hiele's theory in teacher training has also had a positive impact on pre-service mathematics teachers' knowledge of geometry teaching (Usman et al., 2023). To meet students' geometry learning needs, teachers must be able to design appropriate learning activities (Gül-Toker, 2008; cited in Uygün & Güner, 2021). A solid foundation in geometry, along with the teacher's skills in designing and implementing practical lessons, will help students acquire knowledge and progress to higher levels of geometric thinking (Fuys et al., 1988). Therefore, equipping teachers with the necessary professional knowledge to teach geometry is extremely important. According to Yi et al. (2020, 2022), the geometry teaching knowledge of pre-service mathematics teachers is composed of three core components based on van Hiele's theory: (1) knowledge of the mathematical content to be taught - requiring teachers to have a deep understanding of the nature and properties of shapes and their relationships in alignment with the levels of students' cognitive development; (2) knowledge of students' geometric thinking - focusing on the ability to assess and classify students' levels of geometric thinking from visualization to formal deduction, understanding common concepts and errors in students' thinking process; and (3) knowledge of geometry teaching activities - requiring teachers to master teaching methods and tools, design and implement appropriate activities to promote learners' progress through each level of thinking according to the van Hiele model. When used to create instructional activities, the van Hiele model has been shown to positively affect aspiring pre-service teachers' geometric knowledge (Yi et al., 2020; Yi et al., 2022).

4.2. Impacts of van Hiele theory-based instruction on pre-service mathematics teachers' geometric thinking skills

In addition to improving the geometric knowledge of pre-service mathematics teachers, learning phases designed according to the van Hiele model in training pre-service mathematics teachers also promote the development of their geometric thinking levels (Nopriana et al., 2023; Yılmaz & Koparan, 2016). These results are validated through experimental studies with relatively long experimental periods, such as the 14-week experiment of Yılmaz and Koparan (2016). Similarly, studies by Nopriana et al. (2023) demonstrated that after a geometry course taught based on the van Hiele model, the average geometric thinking level of pre-service mathematics teachers improved significantly from the Analysis level to the Formal Deduction level, with many pre-service mathematics teachers achieving the highest level of abstract thinking – Rigor.

Likewise, the van Hiele model of geometric thinking levels, used in designing learning stages and assessment frameworks, also helps lecturers identify and correct misconceptions among pre-service mathematics teachers by promoting their active engagement in learning (Erdogan, 2020; Jupri, 2018; Mbatha & Bansilal, 2023; Mbusi & Luneta, 2021). Qualitative methods have been used in studies applying van Hiele's theory to detect misconceptions among pre-service mathematics teachers through instruments such as written test analysis and in-depth interviews based on the characteristics of geometric thinking levels according to the van Hiele model (Erdogan, 2020; Jupri, 2018; Mbatha & Bansilal, 2023; Mbusi & Luneta, 2021; Mbusi & Luneta, 2023). Additionally, according to Mbusi and Luneta (2023), errors among pre-service mathematics teachers often stem from misunderstandings of the properties of shapes or from calculation and representation mistakes. They can be identified and explained based on the characteristics of van Hiele's thinking levels (Mbatha & Bansilal, 2023). The use of teaching design based on van Hiele's learning stages and the van Hiele model of geometric thinking levels provides a new approach to using the mistakes of pre-service mathematics teachers as a basis for designing learning activities corresponding to each level of thinking, thereby improving their own geometric reasoning ability (Mbusi & Luneta, 2023). In addition, the distinct language and symbol systems of the van Hiele model for each level of thinking require lecturers to use language appropriate to the current level of thinking of pre-service mathematics teachers; otherwise, it can lead to misunderstandings and misconceptions. Using mathematical communication to identify early issues in pre-service teachers' geometric understanding helps lecturers develop their geometric knowledge and skills (Mbusi & Luneta, 2023). In addition, the current deductive thinking skills of pre-service mathematics teachers were also surveyed in a pilot study, based on an analysis of proofs they produced using the van Hiele level thinking model (Jupri, 2018). Consequently, the authors affirm the potential of organizing geometry instruction through lesson plans designed according to the van Hiele model to develop geometric thinking among pre-service mathematics teachers.

4.3. Impacts of van Hiele theory-based instruction on pre-service mathematics teachers' attitudes towards teaching and learning geometry

Besides knowledge and skills, the attitudes of pre-service mathematics teachers towards teaching and learning geometry are also a topic of interest. Numerous studies have shown that implementing a van Hiele-based teaching approach improves pre-service mathematics teachers' attitudes towards geometry (Banson et al., 2022; Mohamed et al., 2025; Usman et al., 2019). This result was supported by a quasi-experimental study by Usman et al. (2019), which compared the attitudes towards geometry of two groups of pre-service mathematics teachers after completing a course taught with a van Hiele model-based approach and a traditional approach. Additionally, specific manifestations of attitudes such as enjoyment, confidence, and awareness of the usefulness of geometry among pre-service mathematics teachers were investigated in Banson et al. (2022), with over 200 pre-service mathematics teachers participating. On the other hand, to enhance students' interest in and understanding of geometry among pre-service mathematics teachers, studies have combined the van Hiele model with technology to create a collaborative, exploratory, and problem-solving learning environment (Mohamed et al., 2025). The study by Mohamed et al. (2025), which combined GeoGebra with learning stages based on the van Hiele model, encouraged pre-service mathematics teachers to actively participate in learning activities such as testing, analysis, practice, comparison, identification, explanation, differentiation, and problem-solving.

5. CONCLUSION

This study synthesizes and analyzes research findings on fostering the geometry knowledge, geometric thinking skills and attitudes towards geometry of pre-service mathematics teachers through applying van Hiele theory-based instruction. The included studies generally report positive impacts of applying the van Hiele model and its learning stages in lesson and assessment design of pre-service mathematics teachers on the reconstruction of geometric

knowledge, fostering knowledge of geometry teaching to develop geometric thinking in students, developing geometric thinking, and nurturing positive learning attitude towards geometry among pre-service mathematics teachers.

The study proposes several recommendations to improve the quality of training pre-service mathematics teachers in teaching geometry including: (1) Reviewing the content, scope and methods of organizing geometry teaching for pre-service mathematics teachers based on the application of van Hiele's theory; (2) Assessing the geometric thinking of pre-service mathematics teachers to design suitable training plans; (3) Improving pre-service mathematics teachers' knowledge and teaching skills in developing their students' geometric thinking; (4) Combining technology tools with teaching stages according to the van Hiele model to improve their geometric knowledge and skills and activate their attitudes towards geometry of pre-service mathematics teachers.

Despite its significance, this study has certain limitations. Firstly, the paper relies primarily on a literature review and lacks an empirical survey to assess the practical impact of van Hiele's theory on geometry competencies of pre-service mathematics teachers. Furthermore, the paper does not analyze factors such as the research context, the duration of the experiment, and differences in sample size to provide more in-depth comparisons and evaluations. Also, this study can not provide an explicit description of the procedures and methods used to apply van Hiele's theory in the training of prospective mathematics teachers. Notably, the high homogeneity of positive outcomes reported across all included studies with the absence of null results also limits the balance of the review.

Based on the results and limitations identified, the study proposes several new directions for future research. Firstly, conduct systematic reviews to thoroughly and comprehensively analyze and synthesize the results of worldwide research on the application of van Hiele's theory in training pre-service mathematics teachers and on its effectiveness. Secondly, conduct experimental studies to verify the effectiveness of van Hiele's theory in developing the geometry teaching competence of pre-service mathematics teachers. Thirdly, future reviews on the topic should actively document and include studies with null or divergent results to provide a balanced and comprehensive understanding of the pedagogical interventions. Besides, there are discussions about challenges in applying van Hiele's theory, including difficulties in determining learners' geometric thinking levels and misconceptions at different learning stages, as well as the role of language in the development of learners' geometric thinking (Mohamed et al., 2025). Therefore, future research could be conducted to elaborate on these discussions and develop methods to overcome the identified challenges.

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